Understanding Long Range Bullets
Part 1: The Nature of Scale

By Bryan Litz

Abstract

When you look at the bullets we use for long range shooting, you can't help but notice the striking similarity between all of them. From .224 thru .30 caliber, they have just about the same proportions. In this article, I'll try to explain the major consequences that scale has on long range bullets. This is the first installment of a two part article. This part establishes some facts about bullet scaling, and the second article examines how you can use the information to make better-informed decisions about your equipment.

Let's be Clear…

This is not meant to be an opinionated rant about why one caliber or bullet is better than another. This is also not about what cartridges are best suited to propel bullets of various sizes. And we're not talking about who makes the best bullets for certain calibers, or specific case studies of 'what works for me'.

What I *am* talking about is the fundamental nature of how BC and stability relates to caliber. The effects of these trends can be subtle, and are often overshadowed by other factors of shooting. But make no mistake; the things I'm discussing are *present* and *true* for all people, places, brands, and on every day of the week. Nature is persistent like that.

How Scaling Affects Area and Volume

I'll apologize up front for the 'physics 101' I'm about to put you thru but I promise, paying attention to the fundamentals early on will pay huge dividends in the understanding of things to come.

When you scale an object in size, it's size changes in all 3 dimensions. The scaling affects the *length,* the *area*, and the *volume*. The interesting thing is that all 3 properties change in different proportions for a given scale factor. The best way to explain this is with an example.

Imagine a square drawn on a piece of paper that's 1" on each side. The square has an area of 1 square inch. Now double the length of each side (you're scaling it by two). The square is now 2"x2", but has an area of $2^2 = 4$ square inches. Furthermore, if you have a cube that's 1"x1"x1" and you double it's scale, the cube is now $2^3 = 8$ cubic inches in size. That's a 4 times increase in area, and an 8 times increase in volume from only a 2 times increase of linear scale. Bottom line; volume increases more than area as you scale an object up. For example, if we scale a bullet by 1.05, its area will increase to $1.05^2 = 1.103$ times it's original area. Furthermore, the volume will increase to $1.05^3 = 1.16$ times it's original volume. In 'geek speak', you would say: "*area* goes up with the *square* of the linear scale, and *volume* goes up with the *cube* of the linear scale".

I'm sure none of this is surprising to those who regularly work with construction materials. You know that it takes more than twice the amount of carpet to cover a 20'x20' floor than a 10'x10' floor. Back to bullets…

The bullets I've chosen to consider for this study are not exact scaled proportions of each other, but they're close. For example, you might note that although the caliber scales up, things like jacket thickness and meplat (bullet tip) diameter stay about the same. It's true, but these are details. As with most of the calculations in this article, my intent is *not* to show hair splitting accuracy, but rather to illuminate some basic and important trends. It's easy to get confused if you try to account for every small detail. *In many cases, understanding the fundamentals can enable you to make an educated guess, which is better than being stumped.*

How Scaling Affects Weight

Now we're going to build on the material in the previous section. Basically, you can re-read the entire explanation on scaling and replace the word 'volume' with the word 'weight'. *Bullet weight* increases in proportion to the third power of the linear scale (there's that 'geek speak' again).

With any scaling exercise, you must start with an original shape to scale. I chose the Sierra 142 gr Matchking as the 'benchmark' for our scaling study. What happens if we take the 142 gr benchmark bullet and scale it's proportions up and down to other calibers? To find out, let's apply what we've discussed about scaling to predict the weight of a 7mm bullet that's proportioned the same as the benchmark 142 gr bullet.

1. Start by figuring out the scale factor:
   7mm/6.5mm = 1.077
2. Now multiply 142 gr by the scale factor 'cubed', like this:
   142gr x 1.077^3 = **177 gr**

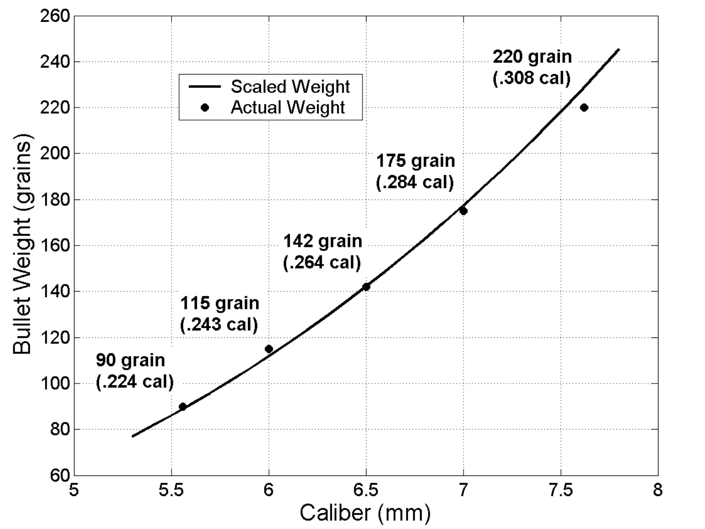
So according to the rules of scaling, a 7mm bullet having the same proportions as the 142 gr benchmark should have a weight of 177 gr. Figure 1 shows the effect of scaling for an entire range of calibers.

At the beginning of this article, I promised to stick to the facts. However, I can't resist a little speculation at this point. Notice how well the bullets scale to the benchmark in terms of weight. That's not an accident. Here's my theory: I suspect that after the 6.5mm, 142 gr Matchking bullet became popular with so many shooters in various disciplines, Sierra decided to design bullets in other calibers to have the same proportions as their 'magic' 6.5mm 142 grainer. With the exception of the 220 gr .308 bullet, all of the other bullets on the trend line in Figure 1 came out after the 142 (I think). Note that the 220 gr bullet is also the farthest from the trend line. OK enough speculation, let's get back to work.

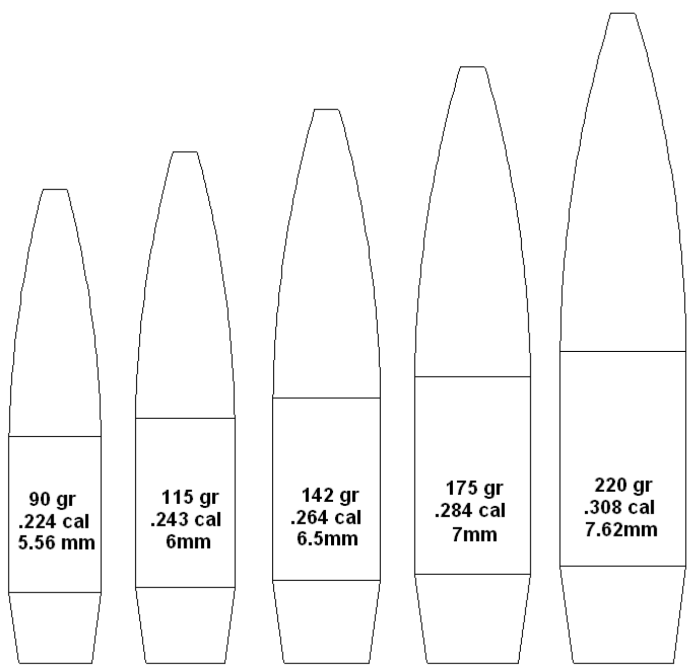
Scaling of the Ballistic Coefficient

It's time to take the next step in our exploration of scaling. So far, we've seen how area and volume change when length is scaled. We've taken a close look at the effects of scaling volume (proportional to weight), and discovered that there are a whole series of Sierra Matchking bullets that are nearly exact scaled copies of each other (by design or by accident). The next logical step is to bring area into the mix.

Scaled Weight Compared to Actual Weight



Predicted Weight scaled from 142 gr bullet				
89 gr	112 gr	142 gr	177 gr	229 gr



Weight of actual bullets (gr)				
90	115	142	175	220

Figure 1. Scaling exact proportions of the benchmark bullet up and down results in calculated weights very close to actual weights.

When you're talking about the shape, mass and cross sectional area of a bullet, you're talking about its ballistic coefficient. The stigma associated with bullet BC suggests that there's something strange and mysterious at work that no one understands. Well, I'm about to show you that even the almighty ballistic coefficient is not beyond comprehension for those who understand the nature of scale. I give you, the equation for ballistic coefficient[1]:

$$BC = \frac{m}{i \cdot d^2}$$

m = mass of the bullet in pounds (weight in grains divided by 7000)

d = diameter of the bullet in inches

i is the 'form factor'

The form factor is a multiplier that relates the drag of any shape to a standard shape. Similar shapes have similar form factors; more on the

[1] For simplicity, this equation does not include the term that accounts for non-standard atmospheric conditions.

form factor later.

For now I'd like to examine the format of the BC equation. Notice how mass is on top, which means increasing mass increases BC (already knew that). Diameter squared is in the denominator, so making a bullet fatter reduces BC (already knew that too). So what does the fancy equation tell me that I didn't already know? Well, you knew the idea, but the equation tells you *how much* of a difference these things make. Knowing about how area and weight scale, we can determine the BC for bullets of neighboring calibers and similar shape.

There are two ways to go about projecting BC's for other calibers. In the first method, call it quick and easy, we assume perfect scaling. In this case, the increase in caliber translates to a scale factor, just like when we scaled the mass in the last section. We'll scale up to 7mm again for BC. The scale factor was 1.077. Since mass is proportional to the cube of the scale factor, and area is proportional to the diameter squared (in the denominator of the BC equation), we can 'scale' the BC from the 6.5mm benchmark (.565)[2] to the 7mm like this:

$$BC = 0.565 \cdot \frac{1.077^3}{1.077^2}$$
$$BC = .565 \cdot 1.077$$
$$BC = 0.609$$

Method 1

And so in the end, you're simply multiplying the original BC by the linear scale factor to get the BC for the other caliber. It's just that easy, or is it?

As with all things, the quick easy method is also the least accurate. The average advertised BC of the 7mm bullet is 0.596 for the speed range of interest (2000 to 2850 fps). Our first method estimated 2.1% high. There's another way that's almost as easy, and a bit more accurate.

In the first method, we assume that the bullet scales perfectly from the benchmark, in which case it would weigh 177 grains. The real 7mm bullet weighs only 175 grains. So a more exact way to calculate BC is by calculating the form factor for the benchmark, and using it for the other bullet. According to the BC equation, and the advertised BC of the benchmark 142 gr bullet (0.565), we calculate a form factor of i = .515. Furthermore, we'll use the actual weight of the 7mm bullet (175 gr) in the BC equation to get a better estimate of the 7mm's BC:

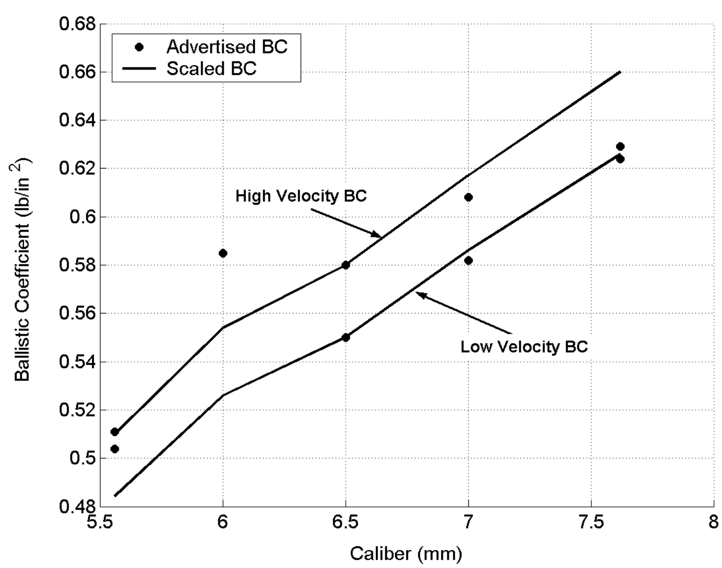
$$BC = \frac{m}{i \cdot d^2} = \frac{(175/7000)}{0.515 \cdot .284^2} = 0.602$$

Method 2

[2] This is the average advertised BC for the velocity range: 2000 fps to 2850 fps. The form factor calculated using this BC is also good in this speed band.

Using method #2, we arrive at an estimate that's only 1.0% different than the advertised BC of the 175 gr 7mm bullet for the same speed range. Figure 2 shows the advertised and calculated BC for a range of calibers based on the principle of scaling (Method 2). These calculated values are compared to advertised BC's for the same bullets. You can see there are two BC's for each caliber. The top one is for high velocity, and the bottom one is low velocity. You can see that the advertised BC's generally fall within the predicted corridor. There are a few things I'd like to point out about Figure 2. First, you might wonder why the 6mm data point is so high. Well, I can think of two reasons why the 6mm might be an outlier.

Ballistic Coefficient Trend



Caliber: mm / inches	5.56 / .224	6.0 / .243	6.5 / .264	7.0 / .284	7.62 / .308
Bullet weight	90	115	142	175	220
Calculated BC	.497	.540	.565	.602	.643
Advertised BC	.502	.585	.565	.596	.627
Percent error	+0.1%	-7.7%	0%	+1.0%	+2.6%

Figure 2. Calculated and advertised BC's.

1. I wasn't able to find a BC for this bullet on the Sierra website like I did the others. I got the one and only value of 0.585 from a different internet resource (Ref 5). The BC for this bullet was only reported for 2850 fps, while the other BC's are averages for velocities between 2000 fps and 2850 fps. So the comparison is not exactly fair.
2. The 6mm bullet has an 11 caliber secant ogive, whereas I think all of the others have a 9 to 9.5 caliber tangent ogive. This means that the 6mm bullet has a different nose shape, like a VLD and has less drag. This would have the effect of lowering the form factor, and elevating the BC.

Both items 1 and 2 above would act to elevate the *reported* BC. Item 1 is an *apparent* increase. I'm sure that at lower velocities, like around 2000 fps, the BC of the 6mm bullet is less than .585, probably averaging about .570 between 2000 and 2850 fps.

Item 2, however, is a *real* reason for the 6mm bullet to have a higher BC. The benefit of the (VLD) secant nose is an advantage at all speeds. In this way, the 115 grain 6mm bullet is an outlier compared to the other bullets in our line up. Another thing you may note is that the BC of the .308 bullet averages at the low end of the band. If you remember when we scaled up the 30 caliber bullet, we found that its weight should be 229 grains. The real bullet is 9 grains lighter than if it were perfectly scaled. If the .30 cal bullet weighed 229 grains, the BC would be .653. That's right smack in the middle of the predicted band. The .244 caliber and 7mm bullets match the BC trend well.

In this way, you are able to use knowledge about trends in scaling to spot when something is out of place, for better or worse. There are names for things that don't follow trends. They're called outliers. Understanding outliers empowers you to choose equipment that has a better than average chance to win. Without an understanding of the fundamental governing trends, you can't spot outliers. You're at the mercy of commercials, old wives tales, and soothsayers.

Take the VLD for example. The 11 caliber secant ogive used on the 6mm bullet is a relatively mild secant ogive, yet the reduction in drag is real, and significant. Consider the 7mm Berger 180 grain VLD with an advertised BC of .682, which is completely off the chart in Figure 2! Even though this is a calculated BC (not a 'fired' BC like Sierra uses), and even though the BC is probably for a higher speed than our average, we can still say the following things about the Berger.

1. At 180 grains, it's 'above the trend line' in terms of weight.
2. With its aggressive, long radius secant ogive nose, you can bet the drag (form factor) reduction is even greater than for the 6mm.

For these reasons we can identify the Berger as a legitimate outlier even if we didn't know what the reported BC was. We would at least know it's worth a look; the US F-Class team is on to it. The VLD design, characterized by the more aggressive secant ogives, has it's own BC trend line that's parallel to the one in Figure 2, but higher. The secant ogive, in general, can reduce the form factor by up to 12%, which increases BC by the same amount.

Before we move on to scaling effects on stability, I feel obligated to say a few words about the dependence of BC on velocity.

The commercial sporting arms industry has universally adopted the G1 drag standard for referencing form factors and ballistic coefficients. Remember the purpose of the form factor? It's to relate the drag of a particular bullet shape to the drag of a standard projectile. Well, there are several standard projectiles to choose from (G1, G5, G7, etc) which may fit certain projectiles better over a wide range of velocity. The G1 drag standard is a compromise for all bullets from .38 caliber pistol bullets up thru shotgun slugs, standard hunting bullets and long-range bullets. Since long range bullets are at the extreme edge of the spectrum in terms of low drag profiles, the G1 standard is actually a poor fit. The consequence of the poor match is that the form factor, and hence the BC is very dependant on projectile velocity[3]. Most of the 'smoke and mirrors' stigma associated with BC's comes from this velocity dependence. At this point, I could to into an entire discussion about Siacci's method, and how trajectories are computed, etc. I'll save that for another day.

One more thing to be careful about with BC is knowing how they're computed. Every company has their own way of figuring BC. Most calculate it, while Sierra actually test fires their bullets to determine BC. All of these methods

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[3] The form factor that relates your projectiles drag to the standard projectile drag is not constant. It's because the two shapes have different 'drag profiles'.

have their pros and cons in terms of accuracy and cost. You should expect typically +-10% error between the BC's reported by different methods. A bullet company may be able to compare their own BC's to each other with great precision, but comparing the advertised BC's from one company to another is a different story. Again, methods to calculate BC's will be left for another day. For now, I hope you're confident enough in your ability to spot outliers, and to make your own comparisons between bullets based on what you know about the consequences of scale.

In part 2, I'll talk about how BC affects wind deflection, and some of the trade-offs involved in chasing the high BC's.

Effects of Scaling on Gyroscopic Stability

Now you're in for it! How can I talk about stability without equations?! Well, I promise this section will be lighter on the math than the last section. You're 'over the hump' now.

The gyroscopic stability factor (Sg) is a measure of how well a bullet is stabilized. In *theory*, Sg only needs to be greater than 1.0 at the muzzle in order to be stable (not tumble) in flight. In *practice*, bullets should have an Sg of at least 1.4 in standard atmospheric conditions to allow for a margin of error. You control the Sg of your bullets by what twist you choose for your barrel. Faster twist gets you a higher Sg. Other factors affecting Sg are: atmospheric conditions; the denser the air,[4] the lower the Sg. Muzzle velocity also affects Sg; faster muzzle velocity increases Sg. Figure 3 shows the gyroscopic stability of our bullets, fired at typical velocities, and how the Sg varies with barrel twist rate.

The one thing that's most apparent about Figure 3 is that the larger caliber bullets require slower twist rates to achieve the same Sg. Why is that? The answer, once again, lies in understanding the nature of scaling.

Two basic things contribute to the gyroscopic stability of spinning bullets:

1. Aerodynamic 'overturning' torque acts to *de-stabilize* the bullet
2. Inertial effects of the spinning mass keep it *stabile.*

As long as the inertial stabilizing effects are stronger than the aerodynamic de-stabilizing effects, the bullet flies point first.

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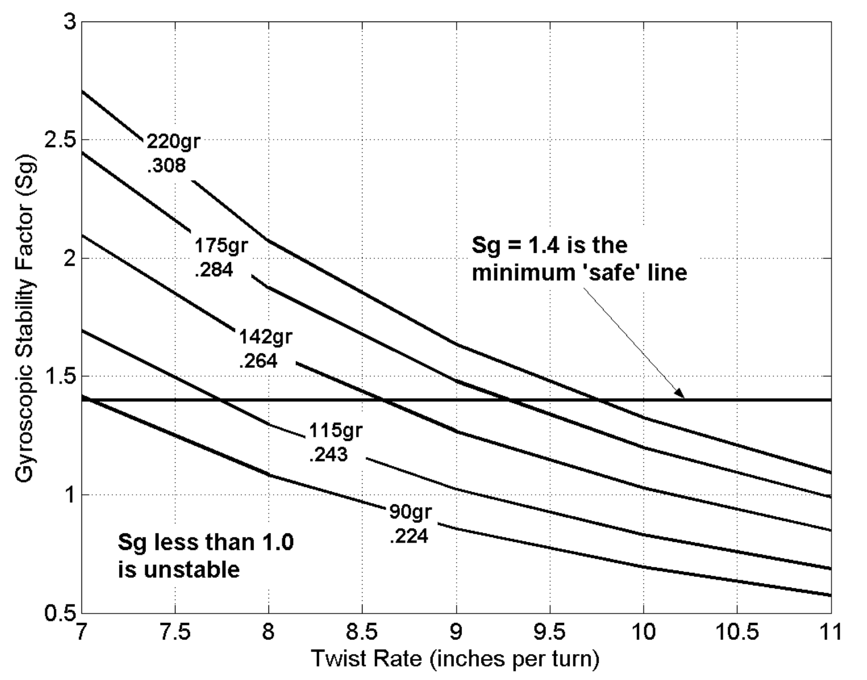
[4] Cold, dry, high pressure air is the most dense

The destabilizing aerodynamic effects are related to the area of the bullet and the separation between the center of gravity and the center of pressure. The stabilizing inertial effects are related more to the mass of the bullet. Does this sound familiar at all? It's the same logic involved in scaling the BC. Increasing the size of the bullet generates more stabilizing inertia than destabilizing aerodynamic torque[6]. The net result is that the larger bullet, fired at the same speed and rate of twist is more stable. Likewise, if you fire a bullet at a higher speed from the same twist barrel, even though the faster bullet has more aerodynamic torque trying to tip it over, the faster bullet is also spinning faster, giving it more stability. The net result is that the bullet is overall more stable when fired at higher velocities.

Effect of Scaling on Stability



Caliber: mm / inches	5.56 / .224	6.0 / .243	6.5 / .264	7.0 / .284	7.62 / .308
Bullet weight (gr.)	90	115	142	175	220
Sg for 1:11" twist	0.57	0.69	0.85	0.99	1.10
Sg for 1:10" twist	0.70	0.83	1.03	1.20	1.33
Sg for 1:9" twist	0.86	1.03	1.27	1.48	1.64
Sg for 1:8" twist	1.09	1.30	1.61	1.87	2.07
Sg for 1:7" twist	1.42	1.70	2.10	2.45	2.71
Minimum twist[5]	**1:7.1**	**1:7.7**	**1:8.6**	**1:9.3**	**1:9.8**

Figure 3. Gyroscopic stability factors were calculated for standard atmospheric conditions. These predicted minimum twist rates may differ slightly from manufacturers suggestions. Manufacturers will typically err on the safe side.

We've noted that the larger bullets require slower twists to be stable, and explained why. Now for the important question: Is there any practical consequence to a faster or slower twist? Tune in next month to find out!

The last thing I'll cover on stability is how it changes with muzzle velocity, and atmospheric conditions. The numbers in Figure 3 are reported for standard atmospheric conditions and for muzzle velocities typical for each round. I didn't look at atmospheric effects for BC, because it's a more trivial problem; you just

[5] Minimum twist required for an Sg=1.4. You may get away with slightly slower twist, but it's not recommended. Higher twist rates are generally ok.
[6] Because mass increases more than area for a given linear scale factor.

multiply the BC by the air density ratio (Ref 4), and presto, you have you're new effective BC. Sg is a little less intuitive because it involves inertial effects as well as aerodynamics. Let's take a look at the benchmark and see how the Sg reacts to some non-standard conditions.

Temp Deg F	Muzzle velocity	Sg
59	2950 fps	1.40
59	3050 fps	1.41
100	3050 fps	1.51
59	2850 fps	1.38
0	2850 fps	1.24

Table 1. Effects of velocity and temperature on Sg.

In Figure 3, we found that the benchmark requires a 1:8.6 twist to achieve the desired Sg of 1.4 at 2950 fps. Table 1 shows what happens to Sg if we keep the 1:8.6 twist and change other conditions. Notice that for higher speeds and temperatures, the Sg goes up. For lower speeds and temperatures, the Sg goes down. Decreasing barometric pressure and increasing humidity also make the air less dense. These things have the same effect as increasing temperature.

Conclusions

In this first part, I've presented some information about the physical consequences of scaling bullets of similar shape thru a range of calibers.

First we looked at scaling effects on BC. Understanding the nature of scaling empowers you to identify trends and outliers, in spite of the 'smoke and mirrors' stigma associated with BC's.

Then we took a look at scaling effects on stability. We found that larger calibers require less twist to be stable than smaller calibers, provided they share common proportions. We finished with a look at how stability is affected by common variables like muzzle velocity and temperature.

A great deal of the information in this first part was academic. Next month, I'll draw on this material to see what the practical consequences are, and how shooters can use the information to make better decisions about the calibers and bullets they choose for long range target shooting.

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