



Maximum Effective Range of Small Arms

By Bryan Litz

In this article, I'll attempt to define a method for finding the maximum effective range of a shooting system under the influence of predefined field variables. This is accomplished using a 6-degree-of-freedom (6 dof) computer simulation that is able to model real world factors influencing the rotation and translation of spin stabilized projectiles. I'll then show how the results can be used to make decisions about what type of rifle is right for a particular application, and how far a weapon may be successfully employed against specific targets.

I'm basically attempting to improve on the antiquated logic that goes something like this: "My rifle can hold a zero, and 1 MOA groups at 100 yards. So if I have an accurate ballistics program indicating drop and drift, I should be able to hit a 10" target at 1000 yards". We all know this logic is flawed, but how, exactly? What are the non-linear effects that prevent accuracy at short range to scale predictably at longer ranges? Read on...

Setting the Stage

First of all, as engineers like to do, I'll start by making up words and assumptions in order to establish the scope of our study. The first term to introduce is the "MER", or "*maximum effective range*" of a weapon system. The MER will be established using a set of MER conditions. For this example, the MER conditions will be chosen to define the MER of a varmint-hunting rifle. The metrics we chose to define the MER of this kind of rifle are: *accuracy* and *lethality* (lethality being a combination of kinetic energy and terminal bullet performance). If the bullet can be successfully delivered to the target with acceptable *accuracy* and *lethality*, the target is said to be within the MER of the shooter.

The rifle that is modeled for this experiment is a .243 caliber rifle with a 1:12 twist, capable of 0.5 MOA at 100 yards using 80-grain varmint bullets,¹ at an average muzzle velocity of 3000 fps. The following MER conditions will be enforced.

Accuracy: Shots must be guaranteed to impact within a 6" circle.

Lethality: at least 500 ft-lbs of kinetic energy at impact.

On our way to setting the stage, we introduce another term: *field variables*. In the present context, field variables refer to all of the things in the field that compound to cause a well-aimed shot to miss the target. Field variables include: misjudgments of wind speed and direction, range uncertainties, variation in muzzle velocity, Coriolis acceleration, uphill/downhill firing, gyroscopic drift, air temperature, humidity and pressure variations, limited precision of sight adjustments, lateral throw-off, aerodynamic jump, etc.

For the present study, the field variables that will be used are:

¹ The Sierra .243 80-grain "Varminter" bullet was used for this example.



1. Left to right (pure cross-) wind is assumed to average 5 mph, with a +2 mph variation. Sights are adjusted to account for the 5 mph prevailing condition.
2. Muzzle velocity averages 3000 fps and has a +-10 fps error (20 fps extreme spread).
3. Firing may occur on any heading at 30 degrees latitude.²
4. Air temperature, humidity, and pressure are known to a degree such that the air density can be calculated to within +-5%.

The final results of the trajectory modeling will show how much each field variable contributes to the overall miss distance.

Let's take a closer look at the field variables. There are several types of field variables. One might choose to separate them by how relevant they are for a particular application. For target shooting at known distance with sighter shots allowed, most of the field variables are irrelevant. For example, when shooting targets, the heading of each shot is the same, likewise with the range and air density. And so for target shooting these things are not variable and will have an identical influence on every shot. In fact for target shooting, the only interesting field variables are wind and muzzle velocity variation. However, a hunting or military application requires that all of the variables be considered due to the combinations that are likely to be encountered in the field.

One cause of dispersion that is not really a field variable is the "*inherent rifle precision*". Inherent rifle precision is easily obtained by observing the grouping potential of the shooting system at short range, before the field variables have a chance to influence dispersion. A 50 or 100-yard group fired from a bench rest or bi-pod in little or no wind is a reliable indicator of inherent rifle precision. Most varmint hunting rifles can be made capable of an inherent rifle precision in the range of 0.5 MOA. 0.5 MOA will be used for the present example.

Up to this point, we've defined our project and scoped its application. Keep in mind that most of the decisions made about MER conditions and field variables are simply assumptions. Different conditions could be chosen to define a different type of MER. There will be more discussion about selecting appropriate MER conditions and field variables later on.

Modeling Exterior Ballistics Using the 6 Degree-Of-Freedom Computer Simulation

So far, MER conditions, field variables, and inherent rifle precision have been put under the microscope. Now let's take a closer look at our system. It doesn't matter what cartridge the .243 caliber rifle is chambered for, or how big the barrel is, or what kind of scope it has, etc. All that matters is that we know it's delivering the 80-grain bullets at an average muzzle velocity of 3000 fps at a twist rate of 1 turn in 12 inches and is capable of 0.5 MOA groups at short range.

The bullets flight, on the other hand, is the most complex and important part of a maximum range analysis. Large amounts of time and effort have gone into

² This is for calculating the Coriolis effect.



searching for suitable tools to calculate the mass properties and aerodynamic coefficients of spin stabilized projectiles.

Aerodynamics and Mass Properties

This section may be considered optional reading for those not interested in the “nitty-gritty” of how the computer simulation works.

The semi-empirical aerodynamic prediction module of the PRODAS code³ has been found to be most suitable for the task of generating aerodynamic coefficients for spin-stabilized projectiles. PRODAS calculates aerodynamic coefficients by applying special curve fits to an empirical database of wind tunnel and firing test results. An empirical method like this is much better at predicting aerodynamics in flight regimes that are governed mostly by viscous effects⁴, especially thru the transonic flight regime. Also, since most bullets share relatively similar configurations, the slight variation of proportions means that the predicted aerodynamics for any bullet will not be too far from the observed aerodynamics of a bullet that was actually tested. It's really a well-suited tool for the application and represents the best aerodynamics prediction tool available (aside from actually performing a wind tunnel test or spark photography analysis of firing tests...\$\$).

The mass properties of the bullet are simpler to calculate than the aerodynamics, but no less important. Mass properties include the bullet mass, center of gravity, axial and transverse moments of inertia (I_{xx} and I_{yy}). These properties are very important to the static and dynamic stability of the bullet in flight. Just like *mass* is proportional to an objects *linear acceleration* due to an applied *force* ($F=ma$), the *moment of inertia* is proportional to an objects *angular acceleration* due to an applied *torque* ($T=I\omega$). We're dealing with a gyroscopically stabilized projectile. It's very important to have an accurate description of the torque (overturning moment applied by the aerodynamics) and the gyroscopic stability (resulting from the bullets spin) in order to model the dynamics of the projectile. Most discussion on the topic of stability is geared toward answering the question “how much spin is needed to stabilize a particular bullet”? This is a very fundamental and important question to answer⁵. However there are other gyroscopic effects on a stable spinning projectile that are also important to quantify. Gyroscopic drift, lateral throw-off, and aerodynamic jump are several “6 degree-of-freedom effects” on a spinning projectile. Now to finally answer the question of how the mass properties are calculated. It's a fairly straightforward procedure of segmenting the bullet into tiny cross-sectional disks, calculating the mass properties of each disk, and adding them all together. While calculating aerodynamic coefficients can be more of an inexact “sum of least squares” fit, calculating mass properties is more like accounting.

³ PROjectile Design Analysis System

⁴ Magnus moment, roll and pitch damping are examples of viscous effects.

⁵ See Don Millers Excellent article in the March 2005 issue of Precision Shooting “A new rule for estimating rifling twist – an aid to choosing bullets and rifles”.

Table 1 shows the mass properties and a truncated table of aerodynamic coefficients used to model the 80-grain varmint bullet.

0.243 Caliber 80-Grain Sierra Varmint Bullet			
Drawing not to scale			
Mass Properties			
Bullet weight	80 grains		
I_{xx}	1.489E-8 SI-ft ²		
I_{yy}	1.135E-7 SI-ft ²		
CG from nose	0.532 in		
Aerodynamics			
MACH	C_{x_0}	Cl_α	Cm_α
1.2	0.467	2.488	2.374
2	0.372	2.870	1.768
2.5	0.330	2.959	1.449
3	0.301	2.899	1.228
Table 1. Geometry, Mass and aerodynamic properties of the 0.243 80-grain varmint bullet.			

All of the mass properties are included in Table 1. However, the complete tables of aerodynamic coefficients are too extensive to include here (see Appendix A). I've chosen to show only 4 of the 16 Mach numbers for each coefficient. The coefficients shown in Table 1 are judged to be the most significant to the stability and flight dynamics of the bullet. C_{x_0} is the "zero yaw" axial force coefficient, akin to the drag coefficient. Cl_α and Cm_α are the slopes of the lift curve and pitching moment coefficients (1/rad) respectively. Aerodynamic coefficients that are used, but not

shown in Table 1 include: Cx^2 (The quadratic dependence of Cx on α), Cmq_Cmad (Pitch damping derivatives), Cnp_α (Magnus moment coefficient derivative) and Cip (Roll damping coefficient). There is much to explore within the realm of aerodynamics, but that's not the focus of this study. Lets continue with modeling the performance.

Procedure for finding the Maximum Effective Range (MER)

Remember that our objective is to find the MER of the system with the given field variables. One way to do this is to choose the field variables randomly, within the defined boundaries, as inputs to the simulation. This would result in a "group" of impact points for a particular range. If none of the shots in the group were further than 3" from center (we defined a 6" diameter circle as the accuracy requirement) then we could conclude that that particular range was within the MER. We would then repeat the process at increasing range until a miss occurred as a result of the field variables, thereby violating the predefined "accuracy" MER condition. Luckily, there is a more efficient way to proceed.

Since all of the field variables have been identified, and the direction of their influence is known, it is possible to conjure "worst case scenarios". The idea is to

find the combination of field variables that work together to result in the greatest deflection in each direction. As an example, let's try to imagine what combination of field variables result in the **highest**, furthest **left** shot possible, and then enter them as inputs into the simulation.

The inputs for the **high left** shot would be (recall the field variables that were identified earlier):

- Wind speed is left to right at 3 mph (Windage adjustment is made for an average speed of 5 mph, but the “wind” field variable is +/- 2 mph) resulting in an impact to the **left** of center.
- Muzzle velocity is average, + 10 fps, or 3010 fps resulting in **higher** impact.
- Shot is fired toward the east (Coriolis acceleration causes **high** shots when shooting to the east, and low shots when shooting west⁶)
- Air density is 5% less than when the rifle was zeroed. Decreased air density results in reduced drag, and a **higher** impact.

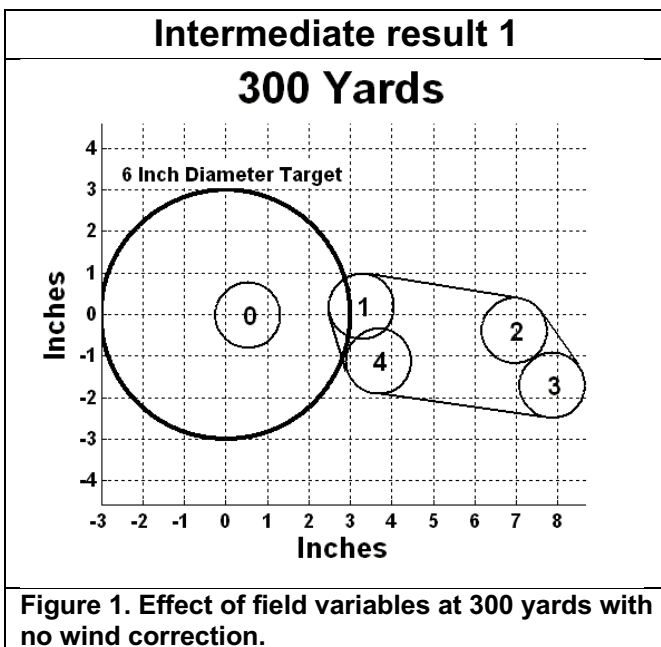


Figure 1. Effect of field variables at 300 yards with no wind correction.

The other 3 extreme corners of the impact area are found in a similar way by adjusting the value of the field variables within their defined bounds. Figure 1 is a target showing the effects of applying the extreme set of field variables to a 300-yard target, and *not* correcting for the average 5mph wind.

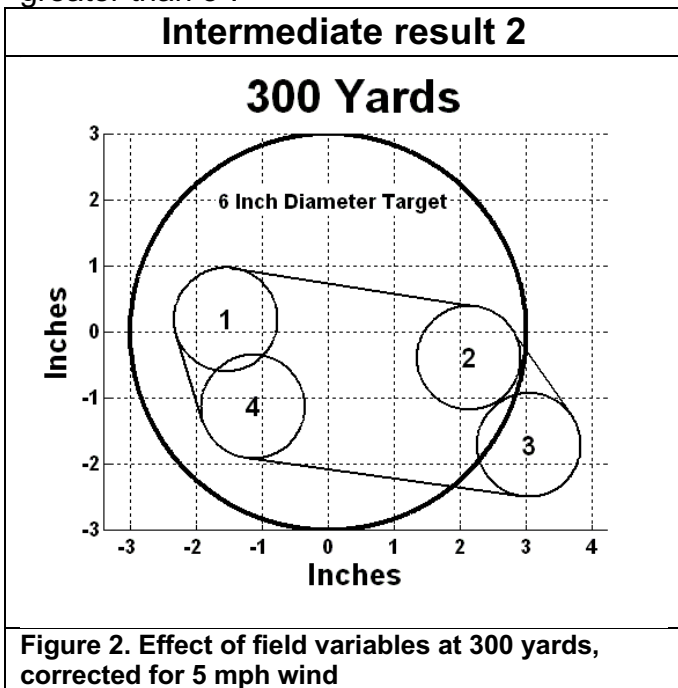
In Figure 1, the circle with the “0” in it represents where the shots hit if elevation is corrected from a 100 yard zero, and no field variables are applied. The circle around the digit represents the inherent accuracy of the rifle (0.5 MOA). Notice that the “0” impact is a little to the right. This small

(0.53”) deflection is a result of gyroscopic drift (0.40”) and Coriolis acceleration (0.13”). These “6 degree of freedom effects” are present no matter what other influences exist. Shots 1-4 are the results of applying the extreme set of field variables listed in Table 2, before the 5 mph average wind is corrected for. According to the 6-dof simulation, the 5 mph wind deflects the bullet 4.82” at 300 yards. After the 4.82” windage correction is made, the impact points in Figure 2 result.

⁶ The horizontal component of the Coriolis acceleration is not a field variable because it is always to the right, its magnitude fixed for given latitudes. However, vertical Coriolis acceleration is a field variable because it depends on the direction of firing. Shooting east causes high shots, shooting west causes low shots.



Figure 2 indicates that 300 yards is beyond the Maximum Effective Range because some of the possible impact area lies outside of the 6" circle (Previously established accuracy MER condition). In other words, the combination of field variables that resulted in shot 3 caused that shot to have a miss distance of greater than 3".



At this point, finding the actual value of the MER is the objective. We know it's less than 300 yards. Unfortunately, there is no way to calculate the MER in one step. It must be done iteratively. That's not as bad as it sounds though because the entire procedure needs not be repeated for the subsequent trial ranges. We know from the 300-yard target that shot #3 is the shot that will define the MER, so shot #3 is the only one we have to iterate for.

Figure 2. Effect of field variables at 300 yards, corrected for 5 mph wind

300 Yards	Extreme Set of Field Variables				Corrected ⁷ 300 Yard Impact		
	Shot number	Wind Speed (mph)	Shooting Direction	Muzzle Velocity (fps)	Air Density (SI/ft ³)	Elev.	Wind.
	1 (high left)	3	East	3010	0.002259	0.23"	-1.55"
	2 (high right)	7	East	3010	0.002259	-0.36"	2.13"
	3 (low right)	7	West	2990	0.002497	-1.68"	3.04"
	4 (low left)	3	West	2990	0.002497	-1.09"	-1.14"

Table 2. Point of impact for each shot subject to the assumed field variables

We can see in Table 2 that shot number 3 impacted 1.68" low, and 3.04" right. Yielding a total miss distance of 3.45". Add to that the 0.25 MOA (radius of grouping ability) that accounts for the inherent rifle precision, and we have a total possible miss distance of 4.24" for the 300-yard range.

⁷ "Corrected" impacts mean that the 300-yard gravity drop, and 5 mph wind drift have been corrected for. The same sight settings were used for every shot at 300 yards. The spread of impact points is due to the field variables.

Table 3 shows the total possible miss distance for other ranges, converging on the MER. Figure 3 shows the effects of the field variables at the calculated MER of 248.5 yards.

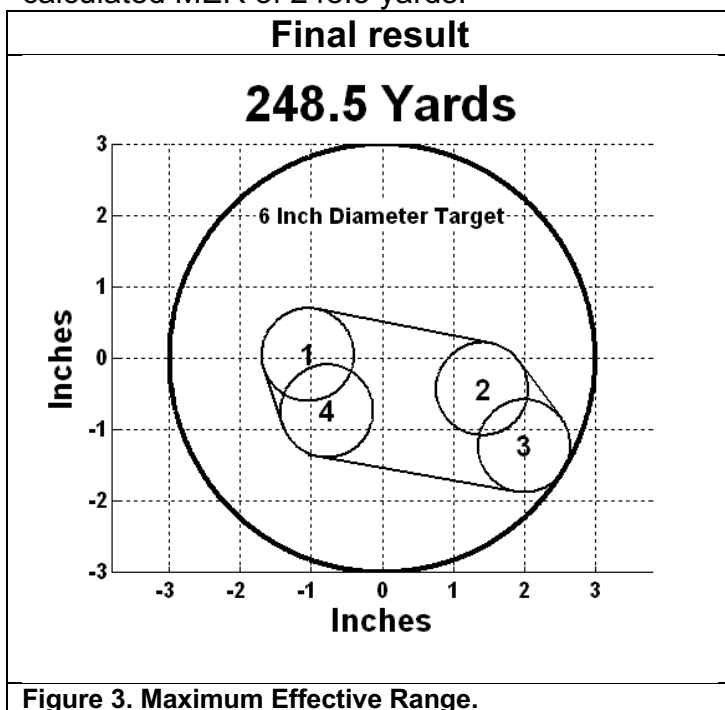


Figure 3. Maximum Effective Range.

Range	Max possible miss distance
300	4.24 inches
200	2.02 inches
250	3.03 inches
248.5	3.00 inches

Table 3. Iterating to find MER

So the *accuracy* MER condition is satisfied at a range of 248.5 yards. At this range, the 80-grain bullet retains 2231 fps (shot #3 conditions), which yields 883 ft-lbs of kinetic energy, thereby satisfying the established 500 ft-lbs MER *lethality* condition. If the bullet had less than the required energy, the MER

would be dictated by decreasing the range until the 500 ft-lbs is satisfied, since the accuracy criteria was already met.

At the beginning of this article, I promised a component breakdown of the effects of each field variable on the trajectory. Table 4 shows the influence of each of the field variables on the original 300-yard target for shot #3.

Contribution to total miss distance Shot #3 at 300 Yards		
Field variable	Elev.	Wind.
Wind speed, Average 5 mph (+2mph error)	-1.01 ⁸ "	2.51"
Shooting direction, West (Coriolis acceleration)	-0.23"	0"
Muzzle Velocity, 2990 fps (-10 fps error)	-0.16"	0"
Air Density, Standard: 0.002378 SI/ft ³ (+5% = 0.002497 SI/ft ³)	-0.29"	0"
Other "6 degree of freedom" effects		
Gyroscopic drift	0"	0.40"
Coriolis acceleration	N/A	0.13"
Total:	-1.68"	3.04"

Table 4. Miss distance component build up. The totals match those listed in Table 2 for shot #3.

Results Analysis

The MER for our system is found to be 248.5 yards. So what? What can that information be used for? I mean, everything's been based on assumptions, and the results are only valid for one particular combination of assumptions, so of

⁸ The vertical component of the wind deflection is due to aerodynamic jump.



what use is the answer “248.5 yards”, really? There are 2 basic ways that the preceding analysis can be applied.

1. Hold the system constant and vary the field variables and/or the MER conditions.
2. Hold the field variables and MER conditions constant and compare different systems (rifle/bullet combinations).

In the first case, one can study the effectiveness of a given shooting system in different applications. For example, lets say that our analysis of the 6mm bullet reflects the common application of someone that hunts varmints in Pennsylvania farm fields. What if that person wanted to take a trip out west to hunt antelope and wonders if the .243 is “enough gun” for the job? We know that the MER of the .243 is 248.5 yards for varmint hunting type field variables, but what is the MER of the .243 for big game hunting in the prairie? The MER conditions and field variables would change. The 6” accuracy requirement can be increased to the vital area of an antelope, maybe 8” or 10”. But the impact energy might need to be increased to 800 or 1000 ft-lbs. That might be hard to do with the 80-grain varmint bullet, so a bigger 6mm bullet is chosen. Maybe a 90 or 100 grain bullet. As you can see, everything changes now that the application is different. You end up with a different MER for big game as for varmints, even though it’s the same rifle. And even with the same application, you can vary the MER conditions and field variables as much as you like to come up with a MER that’s most relevant for the application.

The second case allows “apples to apples” comparison between different rifles for the same application. Lets say the owner of the .243 is considering a new rifle for the same type of varmint hunting. A popular alternative might be one of the fast and flat .22’s like the .220 Swift, or .22-250. In this case, a study would be done on a likely .224 caliber bullet at an expected muzzle velocity to see what kind of MER the system has *with the same field variables and MER conditions as the .243*.

A basic capability for the preceding analysis is offered by most “off-the-shelf” ballistics programs that do a fine job of predicting wind drift and gravity drop. However, I believe a more sophisticated analysis, which captures “6 degree-of-freedom effects”, may be more appropriate when making comparisons and decisions regarding the **Maximum Effective Range** of small arms.

Using the 6-dof simulation to increase Maximum Effective Range

Take another look at Figure 3 and note the impact area of the 4 shots. Your first thought might be “if they were centered, the MER could be extended”. The problem is that none of the available ballistics programs capture the “6 degree of freedom” effects. They all have analytic solvers, which makes them very fast and accurate predictors of gravity drop, drag, and wind drift even for non-standard atmospheric conditions. However, those analytic solvers cannot calculate “6 degree of freedom effects” such as gyroscopic drift, aerodynamic jump, yaw dependant drag, etc. Also, the G1 drag function used in modern ballistics programs is not an accurate drag profile for long boat-tailed bullets. That’s why the B.C. has to be defined piecewise as a function of velocity. The 6-



degree of freedom program uses a numerical solver, which allows the equations of motion to be solved using the actual drag, and not rely on an average fit to a non-representative standard. The problem with the 6-degree of freedom program is speed. It took about 2 minutes for each of the 300-yard trajectories to run on my desktop computer equipped with a 2.08 GHz processor. It's rather impractical to think that a ballistics program running a full 6-degree of freedom simulation can be run on a palm pilot in the field where it's needed. However, there is an alternative...

Run the analytical solution and apply pre-tabulated 6-dof effects to the basic drop and drift results. The whole program could run at practically the same speed and provide corrections resulting in more centered shots. The pre-tabulated 6-dof effects would need to be very specialized for a particular shooting system. For example, drift would depend on twist rate and latitude as well as wind speed and direction. Elevation would depend on wind drift and firing direction as well as muzzle velocity, range, gravity, etc.

This is a very interesting project, and may be the topic of a follow-up article, depending on how well this "modeling and simulation business" is accepted by PS subscribers.

The Horizon

The section on "Results Analysis" explains the immediate relevance of 6 degree-of-freedom trajectory analysis as it applies to the maximum effective range of small arms. I would like to go a step further and suggest some possible, broader applications of the MER idea.

Imagine if the entire sporting arms industry were able to agree on a common set of MER conditions and field variables to use for a particular hunting application. There could be an "agreed-upon" standard to use for every application from prairie dog hunting to big game. Then you could effectively "rack-and-stack" all of the options for a particular application (because it's a true "apples-to-apples" comparison). Right now, the decision of what rifle to choose is mostly made by a combination of experience, advice, and common sense. I'm not hoping to replace these valuable, time proven ideas, but add to them the advantage of detailed engineering analysis.

The "MER paradigm" effectively lays down a yardstick by which to measure the potential value of new products as well. Say, for example, a company markets a new bullet with a higher density core. The bullet can be loaded in any existing standard chambering at the cost of slightly reduced muzzle velocity, and retain much more downrange energy via the elevated B.C.. This will increase the *accuracy* limits of the system. If the bullet exhibits acceptable terminal performance such that the *lethality* MER condition is also met, then the new bullets have real potential for increasing the MER of a shooting system⁹. Realizing that an improvement can be made using such bullets is common sense. However, the MER analysis can show exactly *how much* of a benefit may be realized.

⁹ This assumes, naturally, that the new bullets are manufactured with acceptable quality control.

References:

1. Robert L. McCoy: “Modern Exterior Ballistics”, Schiffer Military History, Atglen, PA, 1999
2. Walter F. Braun: “Aerodynamic Data for Small Arms Projectiles”, BRL Report Number 1630, Jan 1973
3. Don Miller: “A New Rule For Estimating Rifling Twist”, *Precision Shooting*, March 2005, pp 43-48
4. Arrow-Tech Associates: “Projectile Design Analysis - Technical Background”



Appendix A

Complete Table of Aerodynamics for the Sierra 80 grain varmint bullet used in the example

Tabulated Aerodynamics									
MACH	Cx ₀	Cx ²	Cn _α	Cm _α	Cmq+Cm _{αd}	Cnp _α	Cnp _{α3}	Cnp _{α5}	Clp
0.01	0.173	2.271	1.934	2.35	-5.63	-0.433	104.3	-2455.4	-0.012
0.60	0.164	2.271	1.934	2.37	-5.63	-0.433	104.3	-2455.4	-0.012
0.80	0.177	2.764	1.954	2.49	-5.63	-0.315	94.5	-2201.4	-0.011
0.90	0.205	3.227	2.031	2.62	-5.70	-0.109	67	-1333.5	-0.010
0.95	0.269	3.593	2.204	2.69	-6.00	0.073	50.3	-730.3	-0.010
1.00	0.395	4.018	2.305	2.62	-6.50	0.157	36.1	-722.3	-0.010
1.05	0.482	4.441	2.41	2.50	-7.00	0.227	22.3	-502.7	-0.010
1.10	0.477	4.949	2.429	2.43	-8.00	0.256	15.7	-333.4	-0.009
1.20	0.467	5.405	2.488	2.37	-9.00	0.262	11	-220.1	-0.010
1.35	0.446	4.844	2.576	2.29	-10.00	0.274	9.1	-169.3	-0.009
1.50	0.427	4.263	2.683	2.18	-10.57	0.28	8.1	-143.9	-0.009
1.75	0.398	3.699	2.775	1.97	-10.57	0.286	7.1	-118.5	-0.010
2.00	0.372	3.131	2.87	1.77	-10.57	0.292	6.2	-93.1	-0.010
2.50	0.330	2.517	2.959	1.45	-10.57	0.298	5.2	-67.7	-0.010
3.00	0.301	2.077	2.899	1.23	-10.57	0.304	4.2	-42.3	-0.010
4.00	0.259	1.677	2.799	1.27	-10.57	0.304	4.2	-42.3	-0.010

Table A1. Complete table of aerodynamic coefficients used to model the 80 grain Varmint bullet

Definitions and terminology	
α	Total angle of attack in Radians
$Cx = Cx_0 + Cx^{2*}(\sin\alpha)^2$	Axial force coefficient
Cn _α	Normal force coefficient derivative (1/rad)
Cm _α	Overtopping moment coefficient derivative (1/rad)
$Cnp_\alpha = Cnp_\alpha + Cnp_{\alpha3}\alpha^2 + Cnp_{\alpha5}\alpha^4$	Magnus moment coefficient derivative (1/rad)
Cmq+Cm _{αd}	Pitch damping moment coefficient
Clp	Spin decay roll moment coefficient

Table A2. Descriptions of the aerodynamic coefficients used in the computer simulation

